HoTT Worshop 2014

Titles and Abstracts

Speaker: Benedikt Ahrens

Title: Univalent FOLDS

Abstract: The "equivalence principle" of higher category theory says that "meaningful" statements should be invariant under equivalence.

First-Order Logic with Dependent Sorts (FOLDS) was introduced by Makkai as a language for describing higher-categorical structures in which this would always be true, because there is no "equality" that can distinguish equivalent structures. More recently, Homotopy Type Theory (HoTT) is a foundation for mathematics, in which Voevodsky's Univalence Axiom (UA) enforces the equivalence principle for infinity-groupoids by essentially _defining_ "equal" to mean "equivalent".

In previous work, by "relativizing" UA, we defined a notion of "univalent" or "saturated" (1-)category in HoTT that satisfies the principle of equivalence. We now extend this to other higher-categorical structures by defining them à la FOLDS inside HoTT. Any FOLDS-signature comes with a canonical notion of "univalence" for its structures, and such "univalent structures" satisfy the principle of equivalence. Examples include n-categories, dagger-categories, and "doubly weak" double categories.

Speaker: Thorsten Altenkirch

Title: The coherence problem in HoTT

Abstract: Vladimir Voevodsky put forward the challenge to define semisimplicial types in HoTT - it seems that this is impossible due to the coherence problems that arise. Voevodsky put forward a new type
theory HTS based on extensional type theory which distinguishes between types and pretypes. As an alternative we propose a type theory based on intensional type theory which can be easily modelled in existing systems such as Agda. In this type theory as in HTS we can avoid the coherence problem by using a strict equality.

Speaker: Andrej Bauer

Title: How to implement type theory with a reflection rule

Abstract:
I will report on the progress in implementation of a type system HTS0 proposed by Vladimir Voevodsky. The type system is tricky to implement because it has a (strict) equality type which reflects into judgmental equality. This gives us ample opportunities for making mistakes and for derailing the usual algorithms, but it also makes the system extremely expressive. For instance, we can define in it the notion of propositional truncation, or any other higher-inductive type, with the desired computational rules. The plan is to create a system that is useful for experimenting with various concepts in homotopy type theory.

Speaker: Benno van den Berg

Title: Path Object Categories

Abstract: I will discuss categorical models of homotopy type theory. In the first part I will discuss an approach to such models using the notion of "path object category": this provides a fairly simple approach and I will explain that both the category of simplicial sets and the category of cubical sets with connections carries such a structure (jww Simon Docherty). One drawback of this approach is that the syntactic category associated to type theory does not carry such a structure; so I will also discuss a more general approach based on categories with fibrations, which includes the syntactic category as an example. I will talk about a kind of homotopy exact completion for these categories and explain that to construct a model of Aczel's constructive set theory CZF in this exact completion only a very weak form of universe is needed in the original category (jww Ieke Moerdijk).

Speaker: Guillaume Brunerie

Title: The fourth homotopy group of the three-sphere

Abstract : In this talk we will give a proof that the fourth homotopy group of S^3 is Z/2Z. We will see that the proof is entirely constructive and homotopy-theoretic, hence can be carried inside homotopy type theory, and we will discuss the various advantages of having an HoTT-proof and the various challenges in formalizing it.

Speaker: Evan Cavallo

Title: The Mayer-Vietoris Sequence in HoTT

Abstract: We present an essentially topological proof that the Mayer-Vietoris sequence is exact, using the Eilenberg-Steenrod axioms for cohomology. The proof uses squares and cubes, defined as higher inductive types, to simplify computations dealing with higher paths. Adopting a technique used by Dan Licata to prove that the torus is equivalent to $S^1 \times S^1$, we use cube fillers to obtain certain necessary paths, thereby avoiding explicitly calculating these paths and obtaining a dramatically cleaner proof.

The cubical approach to higher paths has also proven generally useful for mechanization in Agda, as the cube type more precisely expresses common path types that arise in proving properties of HITs.

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Speaker: Thierry Coquand

Title: a constructive model of the axiom of univalence

Abstract: We present a variation of the cubical set model, which adds connections and diagonal operations, together with a prototype implementation. Adding connections allows us to interpret the computation rule for the identity elimination as a judgmental equality. By adding diagonals, we also can interpret the two computation rules for circle elimination (on points and on paths) as judgmental equalities.

Speaker: James Cranch

Title: How (not) to define categories in HoTT

Abstract: Work of Ahrens, Kapulkin and Shulman gives a convincing definition of 1-categories inside homotopy type theory. It's certainly natural to ask for much more: a full theory of (infty,1)-categories.
I'll describe a fragment of such a theory, where our categories are those which differ in a finitary way from the (infty,1)-category of types. This fragment contains some handy examples (including all 1-categories), but also fails to capture some pretty crucial examples (such as most (infty,1)-categories of structured types). I'll discuss the strengths and limitations, and hopefully describe some further aspirations.

Speaker: Chris Kapulkin

Title: Type theory and locally cartesian closed quasicategories.

Abstract: Suppose C is a contextual category with Pi- Sigma- and Id- structures (i.e. a categorical model of a type theory admitting rules for Pi-, Sigma-, and Id-types). Then (the underlying category of) C can be equipped with a class of weak equivalences defined syntactically. We may therefore consider the simplicial localization of C, which is the quasicategory obtained by inverting---in the suitable higher-categorical sense---the weak equivalences in C. In this talk, I will outline a proof that the resulting quasicategory is locally cartesian closed.

Speaker: Dan Licata

Title: Cubical Infinite-Dimensional Type Theory

Abstract: In this talk, I will describe work in progress (joint with Guillaume Brunerie) on a cubical syntax for type theory. The goal of the work is to provide a syntactic type theory where the computational aspects of the cubical sets model by Bezem, Coquand, and Huber can be expressed. I will describe a "boundaries-as-terms" cubical type theory, where the basic judgement is "the term u is an n-cube in the type A, together with its boundary", and the cubical operations (faces, degeneracies, symmetries, and diagonals) can be applied to any term. This syntax permits a clean formulation of the computation rules for Kan fillings in Pi, Sigma, and identity types. Moreover, diagonals support the computation rules for higher inductive types. The most salient open issue is how to define the Kan fillings for the universe and for univalence; I would welcome collaboration on this after the talk.

Speaker: Peter LeFanu Lumsdaine

Title: Coherence constructions for dependent type theories

Abstract: Coherence constructions are a vexing technical hurdle which most models of dependent type theory, especially homotopical ones, have to tackle in some way.

I will survey the main known approaches, including the constructions of Hofmann, Voevodsky/Hofmann–Streicher, Lumsdaine–Warren, and Curien–Garner–Hofmann. I will also discuss why we need to worry about coherence in the first place.

Speaker: Egbert Rijke

Title: An algebraic formulation of dependent type theory